Lecture 2: One-Dimensional Kinematics

University Physics V1 (Openstax): Chapter 3
Physics for Engineers & Scientists (Giancoli): Chapter 2

Mechanics, the study of motion, has two parts

- **Kinematics** is the mathematical description of motion. (Our starting point)
- **Dynamics** is the study of the causes of motion (including forces, energy, etc.)

There are 3 types of motion

- **Translational motion** is moving from one location in space to another.
- **Rotational Motion** is changing orientation (direction) without necessarily changing location.
- **Vibrational Motion** is oscillatory motion about an average position.

Translational Kinematics is the Mathematical description of how position changes with time.

- **Position** is given by three variables, the point (x, y, z).
- We use a specific point in each object (or group of objects) called "**the center of mass**".
    - All motion can be described as a combination of translation of the center of mass, rotation about the center of mass, and vibration about the center of mass.
    - This allows us to treat each and **every object as a point**. It doesn't matter what the object is.

    How to find the center of mass will be discussed later in the semester.
- To determine position we must also have a **Reference Frame**, a coordinate axis (mathematics) that is overlaid on top of our reality.
    - There is no fixed way to place a reference frame. You may choose the location of the origin and the directions of the axes. In some cases the origin may even move.
    - While all choices of reference frames are equally valid, some choices can greatly simplify the math needed to solve a problem.
- Each of the position variables must be a function of time.

    $x = x(t)$ $y = y(t)$ $z = z(t)$

One-Dimensional (1D) Translational Kinematics (simplest case with only x, no y or z)

- x = position at time t, the "current position" for whatever time you prefer (a variable).
- x_0 = position at time t=0, the initial position for the reference frame you've chosen (a constant).
    - When looking at the position of the same object at different times (apart from t=0) we will use x_1, x_2, x_3, etc. to denote the position at times t_1, t_2, t_3, etc, respectively.
- $\Delta x = x - x_0$ = **Displacement**.
    - Displacement is a vector. In 1D motion this just means it has a sign attached (+ or –)
    - If x_0 is greater than x, Δx will be negative. (***Displacement can be negative***)

Position (x) and initial position (x_0) are dependent upon your choice of reference frame. Displacement (Δx) is not (at least not until we hit special relativity)

Speed and Velocity (similar but different things)

$$\textbf{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Elapsed Time}} \qquad \textbf{Average Velocity} = \frac{\text{Displacement}}{\text{Elapsed Time}} = v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

- The units of speed and velocity are **m/s** (length per time).

Knowing the units can help identify the quantities in problems.

- Speed (distance) is always positive while velocity (displacement) can be negative.

In NASCAR races the average speed sometimes exceeds 200 mph, but as they return to their starting position (zero displacement) their average velocity is zero.

$$\text{(Instantaneous)}\textbf{Velocity} = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad \text{(Instantaneous) } \textbf{Speed} = |v|$$

- Usually the word "instantaneous" is dropped. It is the default.
- Instantaneous Speed is also always positive while instantaneous velocity can be negative.

Example: Pat is in a race. Assume the track runs along the x-axis with the finish line marked as x = 0 (exact). During a 3.00 s time interval, Pat's position changes from $x_1 = 60.0$ m to $x_2 = 40.5$ m. A) What is Pat's average velocity during this time? B) If he continues to run with the same average velocity, how long does it take him to finish the race?

A) $\Delta x = x_2 - x_1 = 40.5\text{ m} - 60.0\text{ m} = -19.5\text{ m}$ $\qquad v_{avg} = \frac{\Delta x}{\Delta t} = \frac{-19.5\text{ m}}{3.00\text{ s}} = -6.50\ \frac{m}{s}$

B) $v_{avg} = \frac{\Delta x}{\Delta t} \qquad v_{avg} \cdot \Delta t = \Delta x \qquad \Delta t = \frac{\Delta x}{v_{avg}} = \frac{x_f - x_2}{v_{avg}} = \frac{0\text{ m} - 40.5\text{ m}}{-6.50\frac{m}{s}} = 6.23\text{ s}$

Example: A car travels 274 miles from Dallas to San Antonio in 4.50 hours and then turns around and drives 81 miles back from San Antonio to Austin in 1.40 hours. Over this full trip from Dallas to Austin, determine A) the car's average speed, and B) the magnitude of the car's average velocity.

A) $\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Elapsed Time}} = \frac{274\text{ miles} + 81\text{ miles}}{4.50\text{ hours} + 1.40\text{ hours}} = \frac{355\text{ miles}}{5.90\text{ hours}} = 60.2\text{ mph}$

B) $\text{Average Velocity} = \frac{\text{Displacement}}{\text{Elapsed Time}} = \frac{274\text{ miles} - 81\text{ miles}}{4.50\text{ hours} + 1.40\text{ hours}} = \frac{193\text{ miles}}{5.90\text{ hours}} = 32.7\text{ mph}$

Example: To make it to an important interview on time, a driver needs to average 65.0 mph over the 240 miles trip from Dallas to Houston. At the halfway point the driver has only averaged 55.0 mph. How fast does he need to go the rest of the way to reach his appointment on time?

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Elapsed Time}}$$

$$\text{Distance Travelled} = (240\text{ miles})/2 = 120\text{ miles}$$

$$\text{Time (2}^{nd}\text{ half)} = \text{Time (total)} - \text{Time (1}^{st}\text{ half)}$$

$$\text{Average Speed} \times \text{Elapsed Time} = \text{Total Distance Travelled}$$

$$\text{Elapsed Time (first half)} = \frac{\text{Total Distance Travelled}}{\text{Average Speed}} = \frac{120\text{ miles}}{55.0\frac{\text{miles}}{\text{hr}}} = 2.182\text{ hrs} \Rightarrow 2.2\text{ hrs}$$

$$\text{Elapsed Time (whole trip)} = \frac{\text{Total Distance Travelled}}{\text{Average Speed}} = \frac{240\text{ miles}}{65.0\frac{\text{miles}}{\text{hr}}} = 3.692\text{ hrs} \Rightarrow 3.7\text{ hrs}$$

Time (2^{nd} half) = Time (total) – Time (1^{st} half) = 3.692 hrs – 2.182 hrs = 1.510 hrs ⇒ 1.5 hrs

$$\text{Average Speed(2nd half)} = \frac{\text{Total Distance Travelled}}{\text{Elapsed Time}} = \frac{120\text{ miles}}{1.510\text{ hrs}} = 79.47\text{ mph} \Rightarrow 79\text{ mph}$$

You might think that the answer is 75 mph, but this is wrong!

At 75 mph it takes 1.6 hours to cover 120 miles. This would make the total trip time 3.782 hours instead of the desired 3.692 hours. You would be 5 minutes late!

$$\text{Average Speed} = \frac{\text{Total Distance Travelled}}{\text{Elapsed Time}}$$

This is a ratio, and ratios don't work like that!

Suppose during target practice you hit the target on 2 of your first 8 shots, and after a break, you hit it once more on your last 2 shots. Then you hit the target on 3 out of 10 shots. You do NOT get to say that you hit 25% of your shots in the first batch and 50% in the second batch and average these together to say you hit 37.5%

$$\boxed{\frac{2+1}{8+2} = \frac{3}{10} \neq \left(\frac{2}{8} + \frac{1}{2}\right)\frac{1}{2} = \frac{3}{8}}$$

You can only take the average of velocities when they all have the same time interval.

Acceleration **Average acceleration** $= a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$

$$\text{(Instantaneous)}\textbf{Acceleration} = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

- The units of acceleration are m/s^2 (length per time squared).

  Knowing the units can help identify the quantities in problems.

- Acceleration can be positive or negative.
- Usually the word "instantaneous" is dropped. It is the default.
- In one-dimensional motion, when the sign on the acceleration and velocity match, the object is said to be "accelerating" (i.e. its speed is increasing). When the signs on acceleration and velocity differ, the object is said to be "decelerating" (i.e. its speed is decreasing).

Example: A top fuel dragster is capable of accelerating from rest to 160 km/hr (~100 mph) in 0.80 seconds. What is the average acceleration of a dragster that does this?

$$160\frac{km}{hr}\left\{\frac{1000\ m}{1\ km}\right\}\left\{\frac{1\ hr}{60\ min}\right\}\left\{\frac{1\ min}{60\ s}\right\} = 44.444\frac{m}{s} \quad \Rightarrow \quad 44\frac{m}{s}$$

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{\Delta t} = \frac{44.444\frac{m}{s} - 0\frac{m}{s}}{0.80\text{ s}} = 55.555\frac{m}{s^2} \quad \Rightarrow \quad 56\frac{m}{s^2}$$

Example: An object's position is given by x(t) = (5.0 m) + (3.0 m/s)t + (2.5 m/s^2)t^2. Determine the velocity and acceleration as a function of t.

$$v = \frac{dx}{dt} = \frac{d}{dt}\left\{(5.0\ \text{m}) + \left(3.0\frac{\text{m}}{\text{s}}\right)t + \left(2.5\frac{\text{m}}{\text{s}^2}\right)t^2\right\} = \left(3.0\frac{\text{m}}{\text{s}}\right) + \left(5.0\frac{\text{m}}{\text{s}^2}\right)t$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left\{\left(3.0\frac{\text{m}}{\text{s}}\right) + \left(5.0\frac{\text{m}}{\text{s}^2}\right)t\right\} = 5.0\frac{\text{m}}{\text{s}^2}$$

Example: An object accelerates uniformly at 5.0 m/s^2 with an initial velocity of 3.0 m/s and an initial position of 5.0 m. Determine the objects position as a function of time.

$$a = \frac{dv}{dt} \qquad a \cdot dt = dv \qquad \int a \cdot dt = \int dv = v$$

$$v = \int a \cdot dt = at + C = at + v_0 = \left(5.0\frac{\text{m}}{\text{s}^2}\right)t + 3.0\frac{\text{m}}{\text{s}}$$

$$v = \frac{dx}{dt} \qquad v \cdot dt = dx \qquad \int v \cdot dt = \int dx = x$$

$$x = \int v \cdot dt = \int (at + v_0) \cdot dt = \frac{1}{2}at^2 + v_0 t + C = \frac{1}{2}at^2 + v_0 t + x_0$$

$$x = \left(2.5\frac{\text{m}}{\text{s}^2}\right)t^2 + \left(3.0\frac{\text{m}}{\text{s}}\right)t + (5.0\ \text{m})$$

Constant Acceleration (A special case)

- When acceleration is constant, the average acceleration and the instantaneous acceleration are the same ($a = a_{avg}$).
  - $a = a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t}$
- When acceleration is constant, $v_{avg} = \frac{(v + v_0)}{2}$

This is true because the velocity is increasing linearly.
The average of 3, 4, 5, 6, and 7 is just (3+7)/2 = 5.

- We can derive a set of **four equations**.
  - Solve the acceleration equation for v to get the first equation.

$$a = \frac{v - v_0}{t} \qquad at = v - v_0 \qquad at + v_0 = v$$

$$\boxed{\mathbf{v = v_0 + at}}$$

v, a, and t are variables. v_0 is a constant.

  - To get our second equation, solve the equation for average velocity for x and then plug in the previous equation for v_{avg}.

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t} \qquad (v_{avg})t = x - x_0 \qquad (v_{avg})t + x_0 = x \qquad \frac{1}{2}(v + v_0)t + x_0 = x$$

$$\boxed{\mathbf{x = x_0 + \frac{1}{2}(v + v_0)t}}$$

x, v and t are variables. x_0 and v_0 are constants.

- To get our third equation, plug the value of v from the first equation into the second.

$$x = x_0 + \frac{1}{2}\{v + v_0\}t \qquad x = x_0 + \frac{1}{2}\{(v_0 + at) + v_0\}t \qquad x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$\boxed{\mathbf{x = x_0 + v_0 t + \frac{1}{2}at^2}}$$

x, a and t are variables (a is a variable that has been set to a constant value).
x_0 and v_0 are constants.

- To get our fourth (and final) equation, solve the first equation for t, plug that into the 2nd equation, and solve for v^2.

$$v = v_0 + at \qquad v - v_0 = at \qquad t = \frac{v - v_0}{a}$$

$$x = x_0 + \frac{1}{2}(v + v_0)t \qquad x = x_0 + \frac{1}{2}(v + v_0)\left(\frac{v - v_0}{a}\right)$$

$$x - x_0 = \frac{1}{2}(v + v_0)\left(\frac{v - v_0}{a}\right) \qquad 2a(x - x_0) = (v + v_0)(v - v_0)$$

$$2a(x - x_0) = v^2 - v_0^2 \qquad v_0^2 + 2a(x - x_0) = v^2$$

$$\boxed{\mathbf{v^2 = v_0^2 + 2a(x - x_0)}}$$

x, v and a are variables (a is a variable that has been set to a constant value).
x_0 and v_0 are constants.

- There are 4 variables (x, v, a, and t). Each equation has only 3 of them.

$\boxed{\mathbf{v = v_0 + at}}$	v, a, and t $\Rightarrow$ **No x**
$\boxed{\mathbf{x = x_0 + \frac{1}{2}(v + v_0)t}}$	x, v, and t $\Rightarrow$ **No a**
$\boxed{\mathbf{x = x_0 + v_0 t + \frac{1}{2}at^2}}$	x, a, and t $\Rightarrow$ **No v**
$\boxed{\mathbf{v^2 = v_0^2 + 2a(x - x_0)}}$	x, v, and a $\Rightarrow$ **No t**

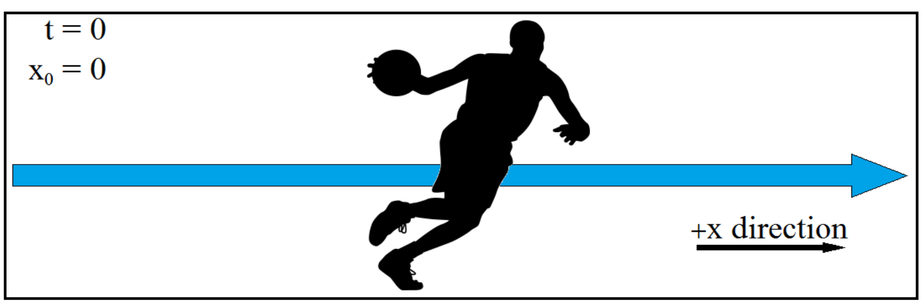
When given the values of two variables and looking for a value of a third, find the equation with all three variables. This is done easily after you determine the 4th variable (missing).

- This set of 4 equations is valid for one object moving with constant acceleration.
    - If acceleration is not constant, you must use the definitions rather than these equations.
    - If there is more than one object moving, you may need two sets of these equations (one for each object).
    - If an object moves with constant acceleration but then suddenly shifts to a new constant acceleration, then you must use a separate set of these four equations for before and after. Usually it's best to treat the before and after as two separate problems.

- Problem solving
    - First, draw a diagram if needed. It can help.
    - Second, set your reference frame (when is t = 0/where is x = 0).
    - Third, extract the data (values of variables and constants) from the problem. Units will help you determine what variables to use for each number.
    - Fourth, determine the formula (or formulas) that you need (i.e. find the path that leads to your answer).
    - Fifth, do the math.

<u>Example</u>: In getting ready to slam-dunk a ball, a player starts from rest and sprints to a speed of 6.0 m/s in 1.5s. Assuming he accelerates uniformly, determine the distance he runs.

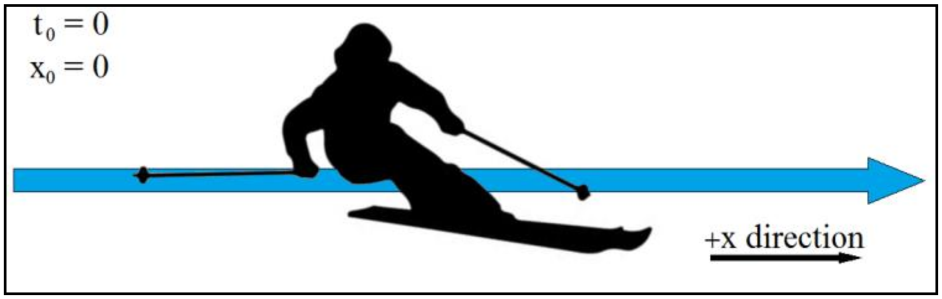
- Draw a diagram.
- Set reference frame.



- Extract data : $x_0 = 0$ $v_0 = 0$ $v = 6.0$ m/s $t = 1.5$s x=?
- Determine formula: "a" is missing $\Rightarrow$ $x = x_0 + \frac{1}{2}(v + v_0)t$
- Do the math: $x = \cancel{x_0}^{\,0} + \frac{1}{2}(v + \cancel{v_0}^{\,0})t = \frac{1}{2}vt = \frac{1}{2}\left(6.0\,\frac{m}{s}\right)(1.5\ s) = 4.5\ m$

Example: (a) What is the magnitude of the average acceleration of a skier who, starting from rest, reaches a speed of 8.0m/s when going down a slope for 5.0s (b) How far does the skier travel in this time?

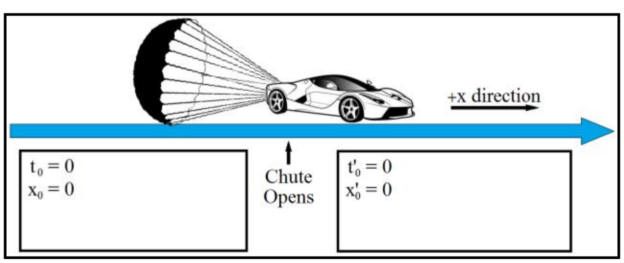
- Part A
  - Draw a diagram.
  - Set reference frame.



  - Extract data : $x_0 = 0$ a=? $v_0 = 0$ v = 8.0 m/s t = 5.0s
  - Determine formula: "x" is missing $\Rightarrow$ $v = v_0 + at$
  - Do the math: $v = \cancel{v_0}^{0} + at = at$ $a = \frac{v}{t} = \frac{8.0\frac{m}{s}}{5.0\ s} = 1.6\frac{m}{s^2}$
- Part B
  - Determine formula: Any formula with "x" will do $\Rightarrow$ $x = x_0 + \frac{1}{2}(v + v_0)t$
  - Do the math: $x = \cancel{x_0}^{0} + \frac{1}{2}(v + \cancel{v_0}^{0})t = \frac{1}{2}vt = \frac{1}{2}\left(8.0\frac{m}{s}\right)(5.0\ s) = 20.\,m$

Example: A drag racer, starting from rest, speeds up for 402 m with acceleration of +17.0 m/s². A parachute then opens, slowing the car down with an acceleration of -6.10 m/s². How fast is the racer moving 350 m after the chute opens?

- Draw a diagram.
- Set reference frame.
  - We have an object where the acceleration changes from once constant value to another.
  - This requires two sets of equations, one before the chute opens and one after.
  - Essentially this is like working two separate problems.
  - We can use two separate reference frames, S = (x,t) for before the chute opens and S' = (x', t') for after.



- Extract data : $x_0 = 0$ $v_0 = 0$ $x = 402$ m $a = 17.0$ m/s^2
  $x'_0 = 0$ $a' = -6.10$ m/s^2 $x' = 350$ m $v' = ?$

- Determine formulas
  - 2nd half with open chute: t' is missing $\Rightarrow$ $v'^2 = v'^2_0 + 2a'(x' - x'_0)$
    - We have a', x' and x'_0, but we don't have v'_0.
    - v'_0 (the initial velocity after the chute opens) is the same as v, the final velocity before the chute opens. So we need to find v (or actually v^2).
  - 2nd half with no chute: t is missing $\Rightarrow$ $v^2 = v_0^2 + 2a(x - x_0)$

- Do the math:

$$v^2 = \cancelto{0}{v_0^2} + 2a(x - \cancelto{0}{x_0}) = 2ax = 2\left(17.0\frac{m}{s^2}\right)(402\ m) = 13{,}668\frac{m^2}{s^2}$$

$$v = \sqrt{13{,}668\frac{m^2}{s^2}} = 116.92\frac{\text{m}}{\text{s}}$$

$$v'^2 = v'^2_0 + 2a'(x' - \cancelto{0}{x'_0}) = v'^2_0 + 2a'x' = 13{,}668\frac{m^2}{s^2} + 2\left(-6.10\frac{m}{s^2}\right)(350\ m) = 9{,}398\frac{m^2}{s^2}$$

$$v' = \sqrt{9{,}398\frac{m^2}{s^2}} = 97\frac{\text{m}}{\text{s}}$$

<u>Example</u>: A driver of a speeding car travelling down a deserted road passes a hidden police car. The driver is moving at a constant speed of 89.5 mph (40.0 m/s) when he passes the police officer. It takes the officer 5.00 seconds to begin pursuit. Once he gives chase he accelerates at a uniform 7.50 m/s^2. What distance does the officer go before he catches the car?

- Draw a diagram. There are three important reference points.
  - 1) The location where the speeding car passes the police car.
  - 2) The location of the speeding car when the police car begins to give chase.
  - 3) The location when the police car catches up to the speeding car.
- Set reference frame.
  - x = 0 is best set at the location where the speeding car passes the police car with the positive x-axis pointing in the direction of motion.

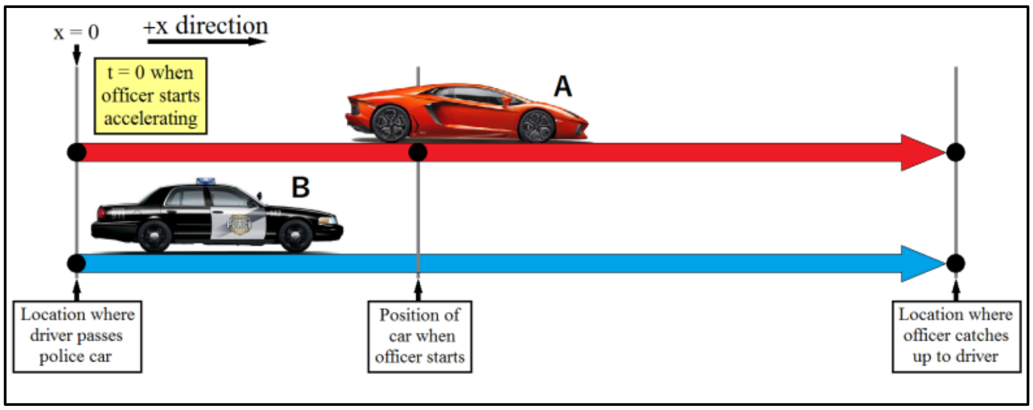
- There are two reasonable options for t = 0.

  1) When the speeding car passes the police car.

     - With this option you must account for the delay of the police car by calculating an artificial x_0 and v_0 that will place him at rest at the origin at t = 5.00s.
     - To get v_0: Set v = 0, a = 7.50 m/s^2, and t = 5.00 s in $v = v_0 + at$
     - To get x_0: Set x = 0, a = 7.50 m/s^2, t = 5.00, and v_0 in $x = x_0 + v_0 t + \frac{1}{2}at^2$

  2) When the police car begins to accelerate.

     - With this option you must calculate the position of the car at t=0, which is rather straight forward.



- Extract data
  - Police car: $x_{B0} = 0$ $v_{B0} = 0$ $a_B = 7.50$ m/s^2
  - Speeding car: $x_{A0} = ?$ $v_{A0} = 40.0$ m/s $a_A = 0$
- Determine formulas
  - To catch up means to be at the same place (x) at the same time (t). Need an equation with both x and t in them. As both accelerations are known, use the equation with x, a, and t (having no v) $\Rightarrow$ $x = x_0 + v_0 t + \frac{1}{2}at^2$
    - Need two versions. One for each vehicle.
    - $x_A = x_{A0} + v_{A0}t + \frac{1}{2}a_A t^2 = x_{A0} + v_{A0}t$
    - $x_B = x_{B0} + v_{B0}t + \frac{1}{2}a_B t^2 = \frac{1}{2}a_B t^2$
  - We also need to determine x_{A0}, which is the distance a car travelling at a constant 40.0 m/s covers in 5.00 s. ($x_{A0} = v_{A0} \cdot \Delta t$)
  - When the police car catches up: $x_A = x_B$

- Do the math:
  - Determine x_{A0}: $x_{A0} = v_{A0} \cdot \Delta t = \left(40.0\frac{m}{s}\right)(5.00\ s) = 200.\ m$
  - Set the distances equal to each other: $x_A = x_B$
  - Plug in formulas for each vehicle and solve for t: $x_{A0} + v_{A0}t = \frac{1}{2}a_B t^2$

$$\frac{1}{2}a_B t^2 - v_{A0}t - x_{A0} = 0$$

$$\frac{1}{2}\left(7.50\frac{m}{s^2}\right)t^2 - \left(40.0\frac{m}{s}\right)t - (200.\ m) = 0$$

$$\left(3.75\frac{m}{s^2}\right)t^2 - \left(40.0\frac{m}{s}\right)t - (200.\ m) = 0$$

  - Quadratic equation is needed:

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$t = \frac{\left(40.0\frac{m}{s}\right) \pm \sqrt{\left(-40.0\frac{m}{s}\right)^2 - 4\left(3.75\frac{m}{s^2}\right)(-200.\ m)}}{2\left(3.75\ \frac{m}{s^2}\right)}$$

$$t = \frac{40.0\frac{m}{s} \pm \sqrt{1600\frac{m^2}{s^2} + 3000\frac{m^2}{s^2}}}{7.50\frac{m}{s^2}} = \frac{40.0\frac{m}{s} \pm \sqrt{4600\frac{m^2}{s^2}}}{7.50\frac{m}{s^2}} = \frac{40.0\frac{m}{s} \pm 67.823\frac{m}{s}}{7.50\frac{m}{s^2}}$$

  - Two roots, one from positive sign (t_1) and one from negative sign (t_2)

$$t_1 = \frac{40.0\frac{m}{s} + 67.823\frac{m}{s}}{7.50\frac{m}{s^2}} = \frac{107.823\frac{m}{s}}{7.50\frac{m}{s^2}} = 14.376\ s$$

$$t_2 = \frac{40.0\frac{m}{s} - 67.823\frac{m}{s}}{7.50\frac{m}{s^2}} = \frac{-27.823\frac{m}{s}}{7.50\frac{m}{s^2}} = -3.710\ s$$

The negative value of time is an extraneous root and can be ignored.

  - Plug t_1 back into distance formulas to determine the answer.

Plugging into both provides an additional check.

$$x_A = x_{A0} + v_{A0}t = (200.\ m) + \left(40.0\frac{m}{s}\right)(14.376\ s) = 775\ m$$

$$x_B = \frac{1}{2}a_B t^2 = \frac{1}{2}\left(7.50\frac{m}{s^2}\right)(14.376\ s)^2 = 775\ m$$

Freely Falling Bodies

- In the absence of air resistance, all objects at (or near) the surface of the Earth accelerate downward at the same rate when released. $|a| = g = 9.80\ m/s^2$
  - g is positive and is sometimes used as a measure of acceleration.

Air force pilots must be able of withstanding a "g-force" of 9g = (9)(9.80 m/s²) = 88.2 m/s²

Example: How long does it take a bowling ball to fall from rest at a height of 10.0 m?

- We'll skip the diagram.
- Set reference frame: Ball dropped at t=0, ground level at y=0, +y-axis pointing up.
- Extract data: $v_0 = 0$ $y_0 = 10.0$ m $y = 0$ $a = -9.80\ m/s^2$ $t = ?$
- Determine formula: v is missing $\Rightarrow$ $x = x_0 + v_0 t + \frac{1}{2}at^2$
- Do the math:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad 0 = x_0 + \frac{1}{2}at^2 \qquad 0 = x_0 - \frac{1}{2}gt^2 \qquad \frac{1}{2}gt^2 = x_0$$

$$gt^2 = 2x_0 \qquad t^2 = \frac{2x_0}{g} \qquad t = \sqrt{\frac{2x_0}{g}} = \sqrt{\frac{2(10.0\ m)}{9.80\frac{m}{s^2}}} = 1.43\ s$$

Would a feather fall at the same rate? What if it was in a vacuum with no air resistance?
In a vacuum with no air resistance, everything falls at the same rate!

Example: A wrecking ball is hanging at rest from a crane when suddenly the cable breaks. The times it takes the ball to fall halfway to the ground is 1.2s. Find the time it takes for the ball to fall from rest all the way to the ground.

- Diagram.
- Set reference frame: Ball dropped at t=0, ground level at y=0, +y-axis pointing up.
- Extract data:
  - At every location: $a = -9.80\ m/s^2$
  - At the top: $t = 0$ $v_0 = 0$ $y_0 =$
  - In the middle: $t_1 = 1.2$ s $v_1 =$ $y_1 = \frac{1}{2} y_0$
  - At the bottom: $t_2 = ???$ $v_2 =$ $y_2 = 0$
- Determine formulas:
  - We know a & y_2 and want to find t_2. v is missing $\Rightarrow$ $y_2 = y_0 + v_{0y}t_2 + \frac{1}{2}at_2^2$
  - $v_0 = 0$ and $y_2 = 0$: $0 = y_0 + \frac{1}{2}at_2^2$

I don't have y_0. How do I get it? Let's look at t_1.

  - We know a & t_2 and want to find y_0. v is missing $\Rightarrow$ $y_1 = y_0 + v_{0y}t_1 + \frac{1}{2}at_1^2$
  - $v_0 = 0$ and $y_1 = \frac{1}{2} y_0$: $\frac{1}{2}y_0 = y_0 + \frac{1}{2}at_1^2$ (with a and t_1 known, this will get us y_0)
- Do the math:
  - Find y_0: $\frac{1}{2}y_0 = y_0 - \frac{1}{2}gt_1^2$ $\qquad \frac{1}{2}y_0 - y_0 = -\frac{1}{2}gt_1^2$ $\qquad -\frac{1}{2}y_0 = -\frac{1}{2}gt_1^2$ $\qquad y_0 = gt_1^2$
  - Use y_0 to get t_2: $0 = y_0 - \frac{1}{2}gt_2^2$ $\qquad \frac{1}{2}gt_2^2 = y_0$ $\qquad gt_2^2 = 2y_0$ $\qquad gt_2^2 = 2gt_1^2$

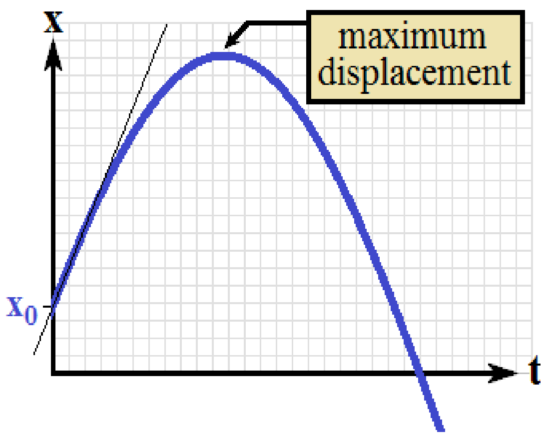
$$t_2^2 = 2t_1^2 \qquad t_2 = \sqrt{2t_1^2} = (\sqrt{2})t_1 = (\sqrt{2})(1.2\ s) = 1.7\ s$$

Graphical Analysis

- Drawing graphs of a variable can offer insights into some problems. It is especially useful on problems where little numerical information is given.

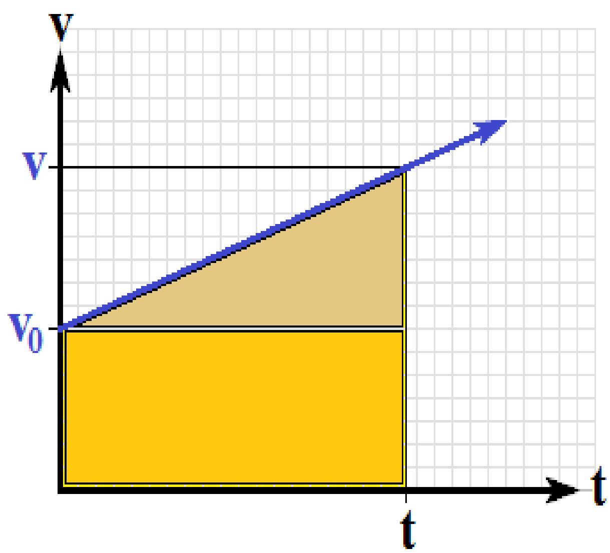
Position graph with no acceleration (a=0)

- $x = mt + b$
- $m = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t} = v$
- Velocity is the slope ($v = v_{avg} = v_0 = \text{const.}$)
- x_0 is the intercept
- $x = vt + x_0$
- When a = 0, position graph is straight lines.



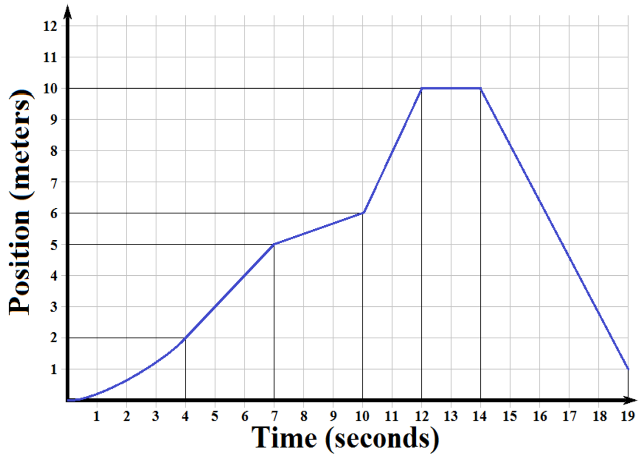
Position graph with const. acceleration (a < 0)

- $x = x_0 + v_0 t + \frac{1}{2}at^2$ (Parabola)
- x_0 is the intercept
- v_0 is the slope at t=0.
- The slope (v) decreases with time (a < 0)
- Maximum displacement occurs when the slope is zero (v = 0)



Velocity graph with constant acceleration (a > 0)

- $m = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t} = a$
- Acceleration is the slope (a = const.)
- Constant acceleration creates straight lines.
- v_0 is the intercept
- $v = at + v_0$
- Rectangle: $LW = v_0 t$
- Triangle: $\frac{1}{2}bh = \frac{1}{2}t(v - v_0) = \frac{1}{2}t(at) = \frac{1}{2}at^2$
- Area under the curve $= \Delta x = v_0 t + \frac{1}{2}at^2$
- The area under the curve is displacement.
- When v is negative (v < 0) so is area ($\Delta x < 0$)



Example:

1) What is x at t = 5? x = 3 m
2) What is v_{avg} between 4 s and 7 s?

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x(7s) - x(4s)}{7s - 4s} = \frac{5m - 2m}{3s} = 1\,m/s$$

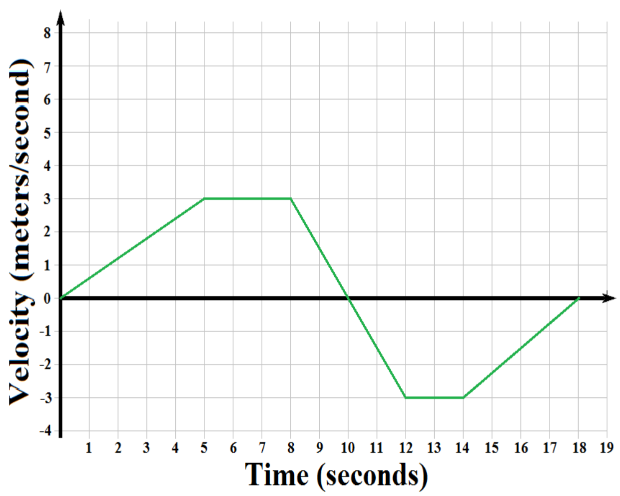
3) What is v at t = 11 s? $v = \bar{v}$ $(10s \le t \le 12s)$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x(12) - x(10)}{12\,s - 10\,s} = \frac{10\,m - 6\,m}{2\,s} = 2.0\,m/s$$

4) What is v at t = 13s? $v = 0$
5) What is a at t = 11s? a=0
6) What is a between t = 0s and t = 4s?

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v(4s) - v(0s)}{4s - 0s} = \frac{1m/s - 0}{4s} = 0.25\,m/s^2$$

Example: From the velocity graph shown determine the following if **x_0=10 m**:

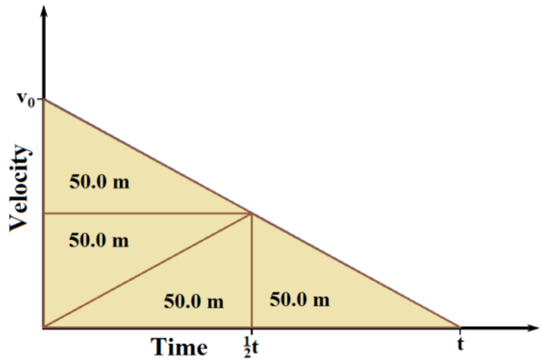


1) What is '**v**' at t = 5?
2) What is '**a**' between 0 s and 5 s?
3) What is '**a**' at t = 6 s?
4) What is '**Δx**' at t = 5s?
5) What is '**x**' at t = 8s?
6) What is '**x**' at t = 14s?

1) $v = 3\ m/s$
2) $a = \frac{\Delta v}{\Delta t} = \frac{v(5s) - v(0s)}{5s - 0s} = \frac{3m/s - 0m/s}{5s} = 0.6\,m/s^2$
3) $a = 0$
4) $\Delta x = ½bh = ½(5s)(3\ m/s) = 7.5\ m$
5) $x = x_0 + \frac{1}{2}bh + LW$ $\quad x = 10\ m + 7.5\ m + (3s)\left(3\frac{m}{s}\right) = 26.5\ m$
6) $x = x(t = 8) + \Delta x$ $\quad x = 26.5\ m + \frac{1}{2}bh - \frac{1}{2}bh - LW$

$x = 26.5\ m + 3m - 3m - 6m = 20.5\ m$

Example: A hockey player passes the puck. After uniform deceleration, the puck comes to rest 200.0 m away. How far does it travel in half this time?



- Draw a graph of velocity
- It starts at v_0 and falls to zero in time t.
- The area under the curve = Δx = 200.0 m
- The distance it covers in half the time is area left of the ½t line.
- We break the area into 4 equal triangles
- The sum of the 4 equal areas must be 200.0 m
- Each triangle is then 50.00 m (a quarter of 200.0 m)
- As 3 triangles make up the area left of the ½t line, the answer is 150.0 m (3 × 50.00 m)

Exercises

1. A car drives 400 miles from Dallas, Texas to Baton Rouge, Louisiana. After 240 miles, the occupants stop for a meal (and gas) in Natchitoches, Louisiana. At that point the car's average speed is 60.0 mph. After spending 30.0 minutes enjoying a relaxing meal in Natchitoches, the occupants return to their car and drive the rest of the way to Baton Rouge at an average speed of 70.0 mph. How long does the entire trip take?

2. After decelerating at a constant rate of 1.50 m/s^2 for 6.00 s, a car is moving at 9.00 m/s (about 20 mph). What was its initial speed?

3. As a car accelerates uniformly from 20.0 m/s to 25.0 m/s as it covers a distance of 112.5 m. Determine the acceleration of the car.

4. A car accelerates uniformly from 20.0 m/s to 25.0 m/s in 4.00 s. What distance does it travel in this time interval?

5. While decelerating at a constant rate of 1.50 m/s^2 for 4.00 s, a car travels 112 m. What was its initial speed?

6. A car starts at rest and accelerates at 2.00 m/s^2 as it travels 100 m. It then decelerates at 2.00 m/s^2 for 5.00s. How far does the car travel from its initial point?

7. A ball is thrown vertically upwards at 10.0 m/s. What is the maximum height the ball attains?

8. A ball is thrown downward from the top of a tall tower at 10.0 m/s, and it strikes the ground 2.00 s later. How tall is the tower?

9. Two cars sit side by side at a stop light. When the light changes both cars start to accelerate. The first car accelerates at $a_1 = \alpha t + \beta$, where $\alpha = 6.00$ m/s^3 and $\beta = 8.00$ m/s^2. The second car accelerates at $a_2 = \kappa t^2+\rho$, where $\kappa = 12.0$ m/s^4 and $\rho = 4.00$ m/s^2. (A) At what time does one car pass the other? (B) How far from the stop light are the cars when they pass? (C) What is the average acceleration of the cars over this interval.

Exercise Solutions

Rather use significant figures as we did in the last exercises, we simply find the answer to 3 significant figures.

1. A car drives 400 miles from Dallas, Texas to Baton Rouge, Louisiana. After 240 miles, the occupants stop for a meal (and gas) in Natchitoches, Louisiana. At that point the car's average speed is 60.0 mph. After spending 30.0 minutes enjoying a relaxing meal in Natchitoches, the occupants return to their car and drive the rest of the way to Baton Rouge at an average speed of 70.0 mph. How long does the entire trip take?

    For the first leg of the journey, the car drives 400 miles at an average speed of 60.0 mph. The time needed to do this can be found using the definition of average speed (just solve for time).

    $$Avg\ Speed = \frac{Distance}{Time} \qquad Time\ \ (T_1) = \frac{Distance_1}{Avg\ Speed_1} = \frac{240\ miles}{60.0\frac{miles}{\cancel{hour}}} \times \frac{1\ hour}{1\ \cancel{hour}} = 4.00\ Hours$$

    Then there was 30.0 minute stop, $T_2 = 0.50\ Hours$

    Then there was the final leg. As the total distance of the trip was 400 miles, and we only went 240 miles in the first two intervals, the distance covered in the 3rd leg is:

    $$Distance_3 = Distance_{Total} - Distance_1 = 400\ miles - 240\ miles = 160\ miles$$

    The time needed for the final leg can be found again using the definition of average speed.

    $$Time\ \ (T_3) = \frac{Distance_3}{Avg\ Speed_3} = \frac{160\ miles}{70.0\frac{miles}{\cancel{hr}}} \times \frac{1\ hr}{1\ \cancel{hr}} = 2.29\ Hours$$

    To get the total time, we just need to add:

    $$Total\ Time = T_1 + T_2 + T_3 = 4.00\ hrs + 0.50\ hrs + 2.29\ hrs = 6.79\ hrs$$

2. After decelerating at a constant rate of 1.50 m/s^2 for 6.00 s, a car is moving at 9.00 m/s (about 20 mph). What was its initial speed?

    In physics, small details can be important. This simple problem isn't as simple as it seems. It will hopefully help you to see the difference between instantaneous velocity (which has a direction given with a sign) and instantaneous speed (which is the absolute value of the velocity). Because we are given speeds, we don't know the signs on any of these values. Because of this, it is important to establish a reference frame. We will assume the car is initially moving in the positive direction, making the initial velocity positive (and hence the same as the speed we want). As the car is decelerating, the acceleration has to be negative (opposite the sign of our initial velocity). Note that 9.00 m/s is the speed after the deceleration. That makes that value the final velocity (v) and not the initial velocity (v_0). However, we can't tell from the problem which sign to give it. The car simply could have slowed down, making v positive. Or it could have come to a stop and started going backwards, making v negative. That means our problem could have two different answers. We'll solve it for both values and see what we get. Let's extract some values:

    $$a = -1.50\frac{m}{s^2} \qquad t = 6.00\ s \qquad v = \pm 9.00\frac{m}{s} \qquad v_1 = -9.00\frac{m}{s} \qquad v_2 = 9.00\frac{m}{s}$$

We are told that it accelerates at a constant rate. So, we can use the formulas for constant acceleration. As you can see, we have time (t), acceleration (a), and velocity (v), and we are looking for the initial velocity (v_0). This doesn't exactly follow the rules you were shown, where you are given two variables and you are looking for a 3rd. There's an easy way to handle problems like these where you are looking for one of the constants. When you are given x and looking for x_0, it's not a whole lot different than when you are given x_0 and you are looking for x. When you are given v and looking for v_0, it's not a whole lot different than when you are given v_0 and you are looking for v. So, if we were given time (t) and acceleration (a), and looking for velocity (v), what equation would you use?

$$v = v_0 + at$$

Solving that for v_0 (by subtracting "at" from both sides) gives us:

$$v_0 = v - at$$

Now let's see what we get for our two values of v:

$$v_{01} = v_1 - at = \left(-9.00\frac{m}{s}\right) - \left(-1.50\frac{m}{s^2}\right)(6.00\ s) = 0\frac{m}{s}$$

$$v_{02} = v_2 - at = \left(9.00\frac{m}{s}\right) - \left(-1.50\frac{m}{s^2}\right)(6.00\ s) = 18.0\frac{m}{s}$$

While both of these are valid solutions, we can rule one out (v_{01}). If our car started at $v_0 = 0$ and ended up at $v = -9.00$ m/s, then our car is actually accelerating. So, our answer is:

$$v_0 = 18.0\frac{m}{s}$$

3. As a car accelerates uniformly from 20.0 m/s to 25.0 m/s as it covers a distance of 112.5 m. Determine the acceleration of the car.

The problem states that the "car accelerates uniformly". That means our equations for constant acceleration are valid. If it isn't clear whether a value is a speed or a velocity, you can generally assume it's a velocity. So, we can take both those values as positive. Those set a direction for our reference frame. We just need an origin. In this case it is easiest to assume that x_0 is zero.

Extract the data: $v_0 = 20.0\frac{m}{s}$ $\quad v = 25.0\frac{m}{s}$ $\quad x_0 = 0\ m$ $\quad x = 112.5\ m$

Find the formula: We have x and v and are looking for a. t was not mentioned.

$v^2 = v_0^2 + 2a(x - x_0)$ *, which as x_0=0, simplifies to:* $v^2 = v_0^2 + 2ax$

Now solve for a. Subtract v_0^2 from both sides. Then divide by 2x.

$$a = \frac{v^2 - v_0^2}{2x} = \frac{\left(25.0\frac{m}{s}\right)^2 - \left(20.0\frac{m}{s}\right)^2}{2(112.5\ m)} = 1.00\frac{m}{s^2}$$

4. A car accelerates uniformly from 20.0 m/s to 25.0 m/s in 4.00 s. What distance does it travel in this time interval?

The problem states that the "car accelerates uniformly". That means our equations for constant acceleration are valid. If it isn't clear whether a value is a speed or a velocity, you can generally assume it's a velocity. So, we can take both those values as positive. Those set a direction for our reference frame. We just need an origin. In this case it is easiest to assume that x_0 is zero.

Extract the data: $v_0 = 20.0\frac{m}{s}$ $v = 25.0\frac{m}{s}$ $x_0 = 0\,m$ $t = 4.00\,s$

Find the formula: We have t and v, and are looking for x. "a" was not mentioned.

$x = x_0 + \frac{1}{2}(v + v_0)t$ *, which as x_0=0, simplifies to:* $x = \frac{1}{2}(v + v_0)t$

$$x = \frac{1}{2}(v + v_0)t = \frac{1}{2}\left[\left(25.0\frac{m}{s}\right) + \left(20.0\frac{m}{s}\right)\right](4.00s) = 90.0\,m$$

5. While decelerating at a constant rate of 1.50 m/s^2 for 4.00 s, a car travels 112 m. What was its initial speed?

The problem states that the car is "decelerating at a constant rate". That means our equations for constant acceleration are valid. If we assume that the initial velocity is positive, then it is also our initial speed. It also makes the acceleration negative as it is decelerating. That sets a direction for our reference frame. We just need an origin. In this case it is easiest to assume that x_0 is zero.

Extract the data: $a = -1.50\frac{m}{s^2}$ $t = 4.00\,s$ $x_0 = 0\,m$ $x = 112\,m$

Find the formula: We have t, x, and a, and we don't have v or v_0. If you look at your four equations, you'll see that v and v_0 are in three of the four. So, if we don't want to solve simultaneous equations, we should use the other equation, which only has v_0.

$x = x_0 + v_0 t + \frac{1}{2}at^2$ *, which as x_0=0, simplifies to:* $x = v_0 t + \frac{1}{2}at^2$

Now solve for v_0. Subtract ½at^2 from both sides. Then divide by t.

$$v_0 = \frac{x - \frac{1}{2}at^2}{t} = \frac{x}{t} - \frac{1}{2}at = \frac{112\,m}{4.00\,s} - \frac{1}{2}\left(-1.50\frac{m}{s^2}\right)(4.00\,s) = 31.0\frac{m}{s}$$

6. A car starts at rest and accelerates at 2.00 m/s^2 as it travels 100 m. It then decelerates at 2.00 m/s^2 for 5.00s. How far does the car travel from its initial point?

In this problem we have an abrupt change from one value of acceleration to another. In these cases, you have to split the problem into two with one acceleration in each. As both have their own separate equations, we can use separate reference frames. In the case we will use the same x coordinate system, but we will have different time coordinates, with t=0 and t'=0 lining up with the start of the acceleration in both cases. We will let x_0 = 0 in the first reference frame.

First part: Extract the Data: $v_0 = 0$ $a = 2.00\frac{m}{s^2}$ $x_0 = 0$ $x = 100\,m$

Second part: Extract the Data: $a' = -2.00\frac{m}{s^2}$ $x'_0 = 100\,m$ $t' = 5.00\,s$ $x' = ?$

In the second part, we have a' and t', and we are looking for x'. v' is not given. That leads us to this equation:

$$x' = x'_0 + v'_0 t' + \frac{1}{2}a't'^2$$

If you compare the data to the equation, you will see that we can't solve for x because we don't have v_0. At this point you need to recognize that the start of the second part of the motion, is the end of the first part of the motion. (v_0' = v). This tells us that we need to solve the first part for v.

In the first part, we have a and x, and we are looking for v. t is not given. That leads us to this equation:

$v^2 = v_0^2 + 2a(x - x_0)$ *, which as $x_0 = 0$ and $v_0 = 0$, simplifies to:* $v^2 = 2ax$.

Take a square root. $v = \sqrt{2ax} = \sqrt{2\left(2.00\frac{m}{s^2}\right)(100\ m)} = 20.0\frac{m}{s}$

As $v = v_0'$, $v'_0 = 20.0\frac{m}{s}$ *, and we can now finish the second part of the problem:*

$$x' = x'_0 + v'_0 t' + \frac{1}{2}a't'^2 = (100\ m) + \left(20.0\frac{m}{s}\right)(5.00\ s) + \frac{1}{2}\left(-2.00\frac{m}{s^2}\right)(5.00\ s)^2$$

$$x' = 175\ m$$

7. A ball is thrown vertically upwards at 10.0 m/s. What is the maximum height the ball attains?

In this problem we have only been given a single value. When that happens, you should start looking for other values that are implied by the problem that aren't explicitly stated. First, let's start by setting our coordinate system. Let's set the ball's release point as x_0=0 and t=0 and upwards as positive. We are given the initial velocity (v_0) and we now have set the initial position (y_0) by choice of our reference frame. We also know the acceleration in free fall, and the final velocity. There can only be one value of v when we reach the maximum height.

Extract the data: $v_0 = 10.0\frac{m}{s}$ $y_0 = 0$ $a = -g = -9.80\frac{m}{s^2}$ $v = 0$

Find the formula: We have v and a, and we're looking for y. "t" isn't offered.

$v^2 = v_0^2 + 2a(y - y_0)$ *, which as $y_0 = 0$ and v = 0, simplifies to:* $0 = v_0^2 + 2ay$

Solve for y by subtracting v_0^2 from both sides and dividing by 2y:

$$y = -\frac{v_0^2}{2a} = \frac{v_0^2}{2g} = \frac{\left(v_0 = 10.0\frac{m}{s}\right)^2}{2\left(9.80\frac{m}{s^2}\right)} = 5.10\ m$$

8. A ball is thrown downward from the top of a tall tower at 10.0 m/s, and it strikes the ground 2.00 s later. How tall is the tower?

Let's set the ground as y=0, the moment the ball is released as t=0 and upwards as positive.

Extract the data: $v_0 = -10.0\frac{m}{s}$ $y = 0$ $a = -g = -9.80\frac{m}{s^2}$ $t = 2.00\ s$

Find the formula: Remember when we have y and are looking for y_0, it usually works the same as if you were given y_0 and were looking for y. So, if we had a and t and were looking for y (v isn't mentioned), what equation would you use?

$y = y_0 + v_0 t + \frac{1}{2}at^2$*, which we can simplify by using y=0 and a = -g. Solving for y_0, we get:*

$$y_0 = -v_0 t + \frac{1}{2}gt^2 = -\left(-10.0\frac{m}{s}\right)(2.00\ s) + \frac{1}{2}\left(9.80\frac{m}{s^2}\right)(2.00\ s)^2 = 39.6\ m$$

9. Two cars sit side by side at a stop light. When the light changes both cars start to accelerate. The first car accelerates at $a_1 = \alpha t + \beta$, where $\alpha = 6.00\ m/s^3$ and $\beta = 8.00\ m/s^2$. The second car accelerates at $a_2 = \kappa t^2 + \rho$, where $\kappa = 12.0\ m/s^4$ and $\rho = 4.00\ m/s^2$. (A) At what time does one car pass the other? (B) How far from the stop light are the cars when they pass? (C) What is the average acceleration of the cars over this interval.

First, note that neither acceleration given is a constant. That means that the equations we derived for constant acceleration are invalid, and we're going to have to use our calculus. Second, "passing" means that the two cars must be at the same place at the same time. That means that we need equations for position for both cars, and we do that by integrating the acceleration twice. Once we get those position equations, will get our answers when we set the two cars positions equal to each other.

$$v_1(t) = \int a_1 dt = \int (\alpha t + \beta) dt = \frac{1}{2}\alpha t^2 + \beta t + C_1$$

This equation is incomplete because we don't have the value of the integration constant, C_1. I used subscripts because we are going to have 4 of them, and in general they won't always be the same. This value can be found if you know the value of v at any moment in time, and we know that it is rest at t=0 (i.e. $v_{10} = v_1(0) = 0$).

$$v_{10} = v_1(0) = \frac{1}{2}\alpha(0)^2 + \beta(0) + C_1 = C_1 = 0 \qquad v_1(t) = \frac{1}{2}\alpha t^2 + \beta t$$

Be advised that this constant C is not always going to be equal to v_0. It is equal to v_0 when a is given by a polynomial because all the terms except C have a power of t in them that all disappear when t=0. When a is some other function you can use the same method, but additional terms will remain in the equation. For example, if you integrate a cosine, you get a sine function, and its value is not zero at t=0. The same holds true for exponential functions.

Next, we integrate the velocity to get the position.

$$x_1(t) = \int v_1 dt = \int \left(\frac{1}{2}\alpha t^2 + \beta t\right) dt = \frac{1}{6}\alpha t^3 + \frac{1}{2}\beta t^2 + C_2$$

Setting the starting point of the two cars as the origin of our system makes $x_{10} = x_1(0) = 0$ and $x_{20} = x_2(0) = 0$. And we can use this to show that C_2 is also zero. Note that while $C_2 = x_0$, this won't always be the case.

$$x_{10} = x_1(0) = \frac{1}{6}\alpha(0)^3 + \frac{1}{2}\beta(0)^2 + C_2 = C_2 = 0 \qquad x_1(t) = \frac{1}{6}\alpha t^3 + \frac{1}{2}\beta t^2$$

Next, we follow the same steps for the second car.

$$v_2(t) = \int a_2 dt = \int (\kappa t^2 + \rho) dt = \frac{1}{3}\kappa t^3 + \rho t + C_3$$

$$v_{20} = v_2(0) = \frac{1}{3}\kappa(0)^3 + \rho(0) + C_3 = C_3 = 0 \qquad v_2(t) = \frac{1}{3}\kappa t^3 + \rho t$$

$$x_2(t) = \int v_2 dt = \int \left(\frac{1}{3}\kappa t^3 + \rho t\right) dt = \frac{1}{12}\kappa t^4 + \frac{1}{2}\rho t^2 + C_4$$

$$x_{20} = x_2(0) = \frac{1}{12}\kappa(0)^4 + \frac{1}{2}\rho(0)^2 + C_4 = C_4 = 0 \qquad x_2(t) = \frac{1}{12}\kappa t^4 + \frac{1}{2}\rho t^2$$

Now that we have equations for x_1 and x_2, we can use them to find the time the cars pass. We just have to set the equations equal to each other and solve for t.

$$x_1(t) = \frac{1}{6}\alpha t^3 + \frac{1}{2}\beta t^2 \qquad x_2(t) = \frac{1}{12}\kappa t^4 + \frac{1}{2}\rho t^2 \qquad \frac{1}{6}\alpha t^3 + \frac{1}{2}\beta t^2 = \frac{1}{12}\kappa t^4 + \frac{1}{2}\rho t^2,$$

I'm going to multiply by 12 to get rid of those fractions and divide by t^2. Then solve the quadratic.

$$2\alpha t + 6\beta = \kappa t^2 + 6\rho \qquad \kappa t^2 - 2\alpha t + 6(\rho - \beta) = 0$$

$$t = \frac{2\alpha \pm \sqrt{4\alpha^2 - 24\kappa(\rho-\beta)}}{2\kappa} \qquad t = \frac{2\left(6.00\frac{\text{m}}{\text{s}^3}\right) \pm \sqrt{4\left(6.00\frac{\text{m}}{\text{s}^3}\right)^2 - 24\left(12.0\frac{\text{m}}{\text{s}^4}\right)\left[\left(4.00\frac{\text{m}}{\text{s}^2}\right) - \left(8.00\frac{\text{m}}{\text{s}^2}\right)\right]}}{2\left(12.0\frac{\text{m}}{\text{s}^4}\right)}$$

$t = \frac{\left(12.00\frac{\text{m}}{\text{s}^3}\right) \pm 36.00\frac{\text{m}}{\text{s}^3}}{24.0\frac{\text{m}}{\text{s}^4}}$ *, and as t can't be negative, we find* t=2.00 seconds, *(The answer to part A)*

To get the position (for part B), we just plug this time into our position equations (both should get the same answer):

$$x_1(2.00\ s) = \frac{1}{6}\left(6.00\frac{\text{m}}{\text{s}^3}\right)(2.00\ s)^3 + \frac{1}{2}\left(8.00\frac{\text{m}}{\text{s}^2}\right)(2.00\ s)^2 = 24.0\ m$$

$$x_2(2.00\ s) = \frac{1}{12}\left(12.0\frac{\text{m}}{\text{s}^4}\right)(2.00\ s)^4 + \frac{1}{2}\left(4.00\frac{\text{m}}{\text{s}^2}\right)(2.00\ s)^2 = 24.0\ m$$

To get the average acceleration (for part C), we need the velocity when they pass. We can get those values by plugging into the velocity equations we got earlier. We know both were at rest at t=0.

$$v_1(2.00\ s) = \frac{1}{2}\alpha t^2 + \beta t = \frac{1}{2}\left(6.00\frac{\text{m}}{\text{s}^3}\right)(2.00\ s)^2 + \left(8.00\frac{\text{m}}{\text{s}^2}\right)(2.00\ s) = 28.0\frac{m}{s}$$

$$v_2(2.00\ s) = \frac{1}{3}\kappa t^3 + \rho t = \frac{1}{3}\left(12.0\frac{\text{m}}{\text{s}^4}\right)(2.00\ s)^3 + \left(4.00\frac{\text{m}}{\text{s}^2}\right)(2.00\ s) = 40.0\frac{m}{s}$$

Then we use the definition of average acceleration, $\bar{a} = \frac{v - v_0}{t}$

$$\bar{a}_1 = \frac{v_1}{t} = \frac{28.0\frac{m}{s}}{2.00\ s} = 14.0\frac{m}{s^2} \qquad \bar{a}_2 = \frac{v_2}{t} = \frac{40.0\frac{m}{s}}{2.00\ s} = 20.0\frac{m}{s^2}$$